

conclude that any frequency dependence is very small. Second, the data show $\Delta\epsilon$ negative so the dielectric constant of the adhesive was less than that of the substrate. It was known in this case that the adhesive had a dielectric constant of 3.25 at low frequencies.

IV. CONCLUSIONS

It has been shown that the wavelength in slot line is sensitive to the dielectric constant and thickness of any adhesive present between the substrate and the conducting surface. If the dielectric constant of the adhesive is less than that of the substrate, wavelength increases and this increase is in direct proportion to the ratio T/D .

While adhesive effect would normally be considered undesirable, it is possible by use of the simple expressions developed here to correct experimental data for comparison with theory without having detailed knowledge of the properties of the adhesive (Fig. 4).

ACKNOWLEDGMENT

The authors wish to thank J. A. Jenners for providing some of the experimental data and Mrs. C. Quinn for her help in preparing the manuscript.

REFERENCES

- [1] S. B. Cohn, "Slot line on a dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 768-778, Oct. 1969.
- [2] —, "Sandwich slot line," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-19, pp. 773-774, Sept. 1971.
- [3] —, "Slot line field components," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-20, pp. 172-174, Feb. 1972.
- [4] T. Itoh and R. Mittra, "Dispersion characteristics of slot lines," *Electron. Lett.*, vol. 7, no. 13, pp. 364-365, July 1, 1971.
- [5] E. A. Mariani, C. P. Heinzman, J. P. Agrios, and S. B. Cohn, "Slot line characteristics," *IEEE Trans. Microwave Theory Tech.* (1969 Symposium Issue), vol. MTT-17, pp. 1091-1096, Dec. 1969.
- [6] G. H. Robinson and J. L. Allen, "Slot line application to miniature ferrite devices," *IEEE Trans. Microwave Theory Tech.* (1969 Symposium Issue), vol. MTT-17, pp. 1097-1101, Dec. 1969.
- [7] E. A. Mariani and J. P. Agrios, "Slot-line filters and couplers," *IEEE Trans. Microwave Theory Tech.* (1970 Symposium Issue), vol. MTT-18, pp. 1089-1095, Dec. 1970.
- [8] R. F. Harrington, *Time Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961.

The Limiting Value of the Interaction Between Symmetrical Fringing Capacitances

HENRY J. RIBLET

Abstract—It is well known that the fringing capacitances determined for rectangular bars between parallel plates interact with each other when $w/(b-t) \rightarrow 0$. The limit of this interaction as $s \rightarrow 0$ for fixed w , b , and t is determined for symmetrical odd-mode fringing capacitances. This limit, together with an exact value known from one rectangular section and the known asymptotic value as $s \rightarrow 0$, permits one to estimate the values for all s . The same is true for the interaction of the symmetrical even-mode fringing capacitances, except that their interaction is readily shown to tend to zero as $s \rightarrow 0$.

If we denote by C_0 the total capacitance of a structure of unit length whose cross section is shown in Fig. 1, then the exact odd-mode fringing capacitance C_{f_0}' is defined by the equation $C_0 = 4C_{f_0}' + 2C_p$ where C_p is the parallel plate capacitance associated with the side of the inner conductor whose length is W_0 . We have then $C_p = 2W_0/(B_0 - T_0)$.

On the other hand, the "approximate" odd-mode fringing capacitance, $C_{f_0}'^1$, is defined as half the limit of the difference between the total capacitance and the parallel plate capacitance of the structure, shown in Fig. 2, as the magnetic wall recedes to infinity at the right.

If we denote $C_{f_0}' - C_{f_0}'^1$ by $\Delta C_{f_0}'$, then this short paper is concerned in a general way with the evaluation of $\Delta C_{f_0}'$ for given W_0 , B_0 , and T_0 as a function of S ; and, in particular, with the value of $\Delta C_{f_0}'$, the limit of $\Delta C_{f_0}'$ as $S \rightarrow 0$. The special interest in the value of $\Delta C_{f_0}'$ arises from the fact that exact values of $\Delta C_{f_0}'$ are already known for $S = \infty$ and at one intermediate point [2]. From an accurate estimate for $\Delta C_{f_0}'$, one may then determine the value of C_{f_0}' . This quantity un-

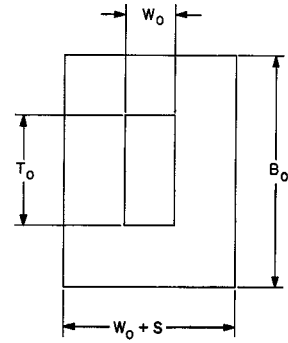


Fig. 1. Geometry defining C_{f_0}' .

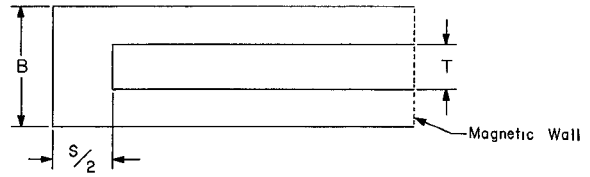


Fig. 2. Geometry defining $C_{f_0}'^1$.

doubtedly is more accurate for general design purposes than C_{f_0}' , since it is exact in the symmetrical case regardless of any interaction.

It is not difficult, following Bowman [3], to express the quantities B , S , and T of Fig. 2, except for a scale factor, in terms of two independent real parameters, a and k , where k is the modulus of the Jacobi elliptic functions involved. It is no restriction to assume that $0 \leq k \leq 1$ and that $0 \leq a \leq K$. Then we have

$$B = 2K' \left\{ \frac{\text{sn } a \text{ dn } a}{\text{cn } a} - Z(a) \right\} - \frac{\pi a}{K} + \pi \quad (1)$$

$$S = 2K \left\{ \frac{\text{sn } a \text{ dn } a}{\text{cn } a} - Z(a) \right\} \quad (2)$$

$$T = 2K' \left\{ \frac{\text{sn } a \text{ dn } a}{\text{cn } a} - Z(a) \right\} - \frac{\pi a}{K} \quad (3)$$

The approximate odd-mode fringing capacity, $C_{f_0}'^1$, for this geometry is given in terms of the same parameters, a and k , by the expression²

$$\pi C_{f_0}'^1 = 2(K - a) \left\{ \frac{\text{sn } a \text{ dn } a}{\text{cn } a} - Z(a) \right\} - 2 \log(k \text{ sn } a \text{ cn } a) - 4 \log(\theta_n(a)). \quad (4)$$

Here the functions are all those which are familiar from Jacobi's theory of elliptic functions, but it may be well to recall that $\theta_n(a) = \Theta(a)/\Theta(0)$.

It is clear from (2) that $S \rightarrow 0$ as $a \rightarrow 0$. If now T/B is to approach the finite limit, T_0/B_0 , as $a \rightarrow 0$, then T must approach a finite limit > 0 as $a \rightarrow 0$. This can only happen if $K' \rightarrow \infty$ and, in turn, $k \rightarrow 0$. We determine then the limit of $C_{f_0}'^1$ as a and $k \rightarrow 0$; and, for this, we will need the precise relationship of a and k in this limit.

To this end, we write down the expansions of the various elliptic quantities occurring in (1), (2), and (3) in ascending powers of a and k . Thus

$$K = \frac{\pi}{2} \left(1 + \frac{k^2}{4} + \dots \right)$$

$$K' = \left(1 + \frac{k^2}{4} + \dots \right) \log \frac{4}{k} - \frac{k^2}{4} + \dots$$

$$\text{sn}(a, k) = a - \frac{1+k^2}{6} a^3 + \dots$$

Manuscript received December 11, 1972; revised June 6, 1973.
The author is with Microwave Development Laboratories, Inc., Needham, Mass. 02194.

¹ This is equivalent to the $C_{f_0}'^1$ used by Getsinger [1], and it should also be noted that the geometrical capacitances of this short paper must be multiplied by the permittivity to obtain the true capacitances.

² This formula is somewhat simpler than the one given by Getsinger [1], to which it may be presumed to be equivalent on the basis of a comparison of numerical results.

$$\begin{aligned} \text{cn}(a, k) &= 1 - \frac{a^2}{2} + \dots \\ \text{dn}(a, k) &= 1 - \frac{k^2}{2} a^2 + \dots \\ Z(a, k) &= \left(\frac{k^2}{2} + \dots\right)a - \left(\frac{k^2}{3} + \dots\right)a^3 + \dots \end{aligned} \quad (5)$$

When these series are substituted in (1), (2), and (3) and the limits found as a and $k \rightarrow 0$, one may neglect all but the first term in the last four equations. Then, if the scale factor, $(B_0 - T_0)/\pi$, is introduced to maintain the required spacing between the parallel plates of Fig. 2, the values of B , S , and T become

$$\pi B' = 2(B_0 - T_0) \left\{ \frac{\pi}{2} + \left(\log \frac{4}{k} - 1 + O(k) \right) a \right\} + O(a^3) \quad (6)$$

$$\pi S' = (B_0 - T_0) \pi (1 + O(k)) a + O(a^3) \quad (7)$$

$$\pi T' = 2(B_0 - T_0) \left(\log \frac{4}{k} - 1 + O(k) \right) a + O(a^3) \quad (8)$$

where $O(x)$ denotes quantities such that $O(x)/x$ is bounded as $x \rightarrow 0$. Similarly, in the vicinity of a and $k = 0$,

$$C_{f_0}' = -\frac{2}{\pi} \log(ak) + O(a) \quad (9)$$

since $\log(1 + O(x)) = O(x)$.

Having fixed $B' - T' = B_0 - T_0$ by our choice of the scaling factor, we can now determine how $\log k \rightarrow \infty$ as $a \rightarrow 0$ by considering the condition imposed by the requirement that $T' = T_0$ as a and $k \rightarrow 0$. Thus, for small a and k ,

$$\frac{T_0}{(B_0 - T_0)a} = \frac{2}{\pi} \left(\log \frac{4}{k} - 1 \right) + O(k) + O(a^2). \quad (10)$$

We see that k vanishes exponentially with $1/a$ so that $O(k)$ is certainly $O(a)$. Solving (10) for $\log(k)$ and substituting in (9) we find

$$C_{f_0}' = \frac{T_0}{(B_0 - T_0)a} - \frac{2}{\pi} (\log(a) - 1 + \log 4) + O(a). \quad (11)$$

It is convenient to express this in terms of the plate spacing, g . From (7) $g = S'/2 = (B_0 - T_0)a/2 + O(a^2)$. Thus, for small g ,

$$C_{f_0}' = \frac{T_0}{2g} - \frac{2}{\pi} \left\{ \log \left(\frac{2g}{B_0 - T_0} \right) + \log 4 - 1 \right\} + O(g). \quad (12)$$

It should be observed that the term $T/2g$ is just half of the capacitance of a capacitor of width, T_0 , plate spacing, g , and unit length.

Now it is also possible to determine the limiting value of the exact odd-mode fringing capacitance, C_{f_0} , as $S \rightarrow 0$ by considering the problem from a different point of view. If we apply the formulas for B , S , and T of Fig. 2 to the dimensions of Fig. 1 after it has been rotated through 90° , then, denoting the approximate odd-mode fringing capacitance by \bar{C}_{f_0}' , it is readily seen that \bar{C}_{f_0}' approaches the exact odd-mode fringing capacitance, \bar{C}_{f_0} as $B \rightarrow T$, since the interaction between the fringing capacities then approaches zero. Now, using the fact that the total capacity of the structure must be the same no matter how it is viewed, we have

$$C_{f_0} + \frac{W_0}{B_0 - T_0} = \bar{C}_{f_0}' + \frac{T_0}{S_0} \quad (13)$$

and so the limiting value of C_{f_0} can be found from the limiting value of \bar{C}_{f_0}' .

To find the limiting value of \bar{C}_{f_0}' , we again turn to (1)–(4) but now determine the relationship between a and k so that, after a suitable scale factor has been selected, while $T' = W_0$ and $S' = B_0 - T_0$, $B' \rightarrow W_0$. If we select W_0/T as the scale factor and choose for k the value, k_0 , for which

$$K'/K = W_0/(B_0 - T_0) \quad (14)$$

and permit $a \rightarrow K(k_0)$, we readily find from (1)–(3) that, while $T' = W_0$, $S' \rightarrow B_0 - T_0$ and $B' \rightarrow W_0$. This path in the (a, k) plane defined by $(a \rightarrow K, k_0)$ which terminates in (K, k_0) will be referred to as the approximating path. Clearly, the ideal path in the (a, k) plane along which $B' \rightarrow W_0$ while $S' = B_0 - T_0$ must terminate in the same point (K, k_0) because of the continuity of the equations defining B' , S' , and T' in terms of a and k . Thus after showing that the limiting

value of \bar{C}_{f_0}' does not depend on the particular path chosen,³ we shall have found the correct limiting value of \bar{C}_{f_0}' when we have found its limiting value on the approximating path. Thus we may find the limiting value of \bar{C}_{f_0}' by evaluating (4) in the limit as $a \rightarrow K$ after k has been given the value k_0 .

To this end, it is convenient to write $a = K - u$ and to consider the behavior of the various functions involved as $u \rightarrow 0$. It is readily found that

$$\begin{aligned} \text{sn } a &= \text{sn}(K - u) = \frac{\text{cn } u}{\text{dn } u} \\ \text{cn } a &= \text{cn}(K - u) = k' \frac{\text{sn } u}{\text{dn } u} \\ \text{dn } a &= \text{dn}(K - u) = \frac{k'}{\text{dn } u} \\ Za &= Z(K - u) = -Zu + \frac{k^2 \text{sn } u \text{cn } u}{\text{dn } u} \\ \theta_n a &= \theta_n(K - u) = \text{dn } u \frac{\theta_n u}{\sqrt{k'}} \end{aligned} \quad (15)$$

then, using (5) we have

$$\begin{aligned} \frac{\text{sn } a \text{ dn } a}{\text{cn } a} &= \frac{1}{u} \left(1 + \frac{2k^2 - 1}{3} u^2 + \dots \right) \\ \text{sn } a \text{ cn } a &= k' u \left(1 - \left(\frac{2}{3} - \frac{5}{6} k^2 \right) u^2 + \dots \right) \\ Za &= \left(\frac{k^2}{2} + \dots \right) u - \left(\frac{k^2}{3} + \dots \right) u^3 + \dots \\ \log(\theta_n a) &= -\frac{1}{2} \log k' + O(u^2) \end{aligned} \quad (16)$$

so that, for small u , independently of k ,

$$\begin{aligned} B' &= W_0 \frac{B}{T} = W_0 + \frac{W_0 \pi}{2K'} u + O(u^2) \\ S' &= W_0 \frac{S}{T} = W_0 \frac{K}{K'} + O(u) \\ T' &= W_0. \end{aligned} \quad (17)$$

We see that for $k = k_0$, B' , S' , and T' have the required behavior as $u \rightarrow 0$.

Substituting (16) in (4) we find, independently of k , for small values of u ,

$$\pi \bar{C}_{f_0}' = 2 - 2 \log(kk'u) + 2 \log k' + O(u). \quad (18)$$

Clearly, as $u \rightarrow 0$, the limiting value of \bar{C}_{f_0}' is independent of the precise manner in which $k \rightarrow k_0$. Thus in (18) we may simply replace k by k_0 and then allow $u \rightarrow 0$. Then also in (17) it is no restriction to assume that $k = k_0$.

The decreasing quantity $(B' - T')/2$ was denoted earlier by g . Thus for small u ,

$$g = \frac{W_0 \pi}{4K'} u + O(g^2). \quad (19)$$

Then, solving for u in terms of g and substituting in (18), we have

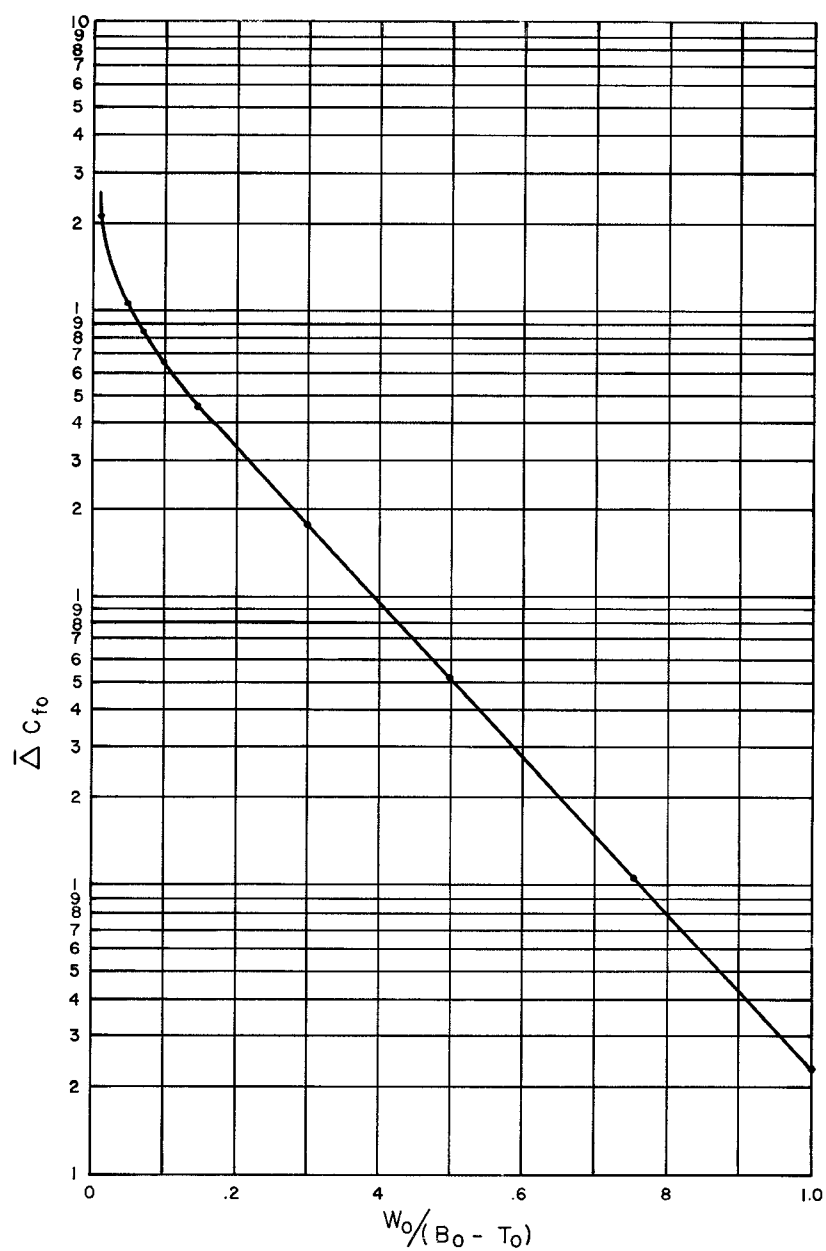
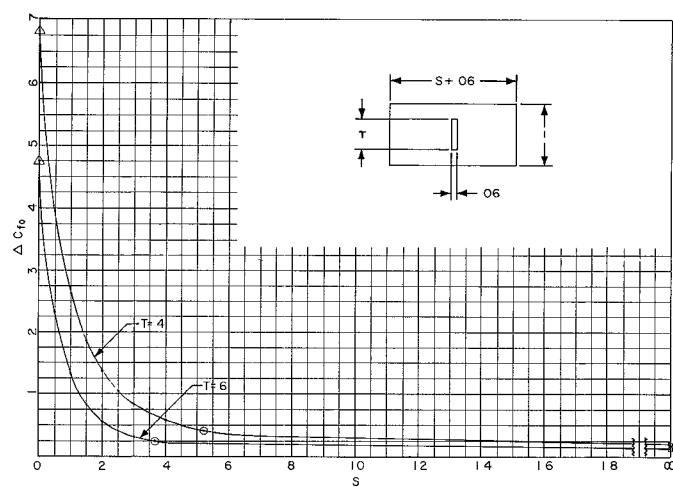
$$\bar{C}_{f_0}' = \frac{2}{\pi} \left\{ 1 - \log k_0 - \log \frac{4K'g}{\pi W_0} \right\} + O(g). \quad (20)$$

Equation (13) is valid for all values of S and, in particular, when $S = 2g$. Thus from (12) and (13), for small g ,

$$\begin{aligned} \Delta C_{f_0} &= C_{f_0}' - C_{f_0} = -\bar{C}_{f_0}' + \frac{W_0}{B_0 - T_0} \\ &\quad - \frac{2}{\pi} \left\{ \log \frac{2g}{B_0 - T_0} + \log 4 - 1 \right\} + O(g). \end{aligned} \quad (21)$$

Then adding and subtracting \bar{C}_{f_0}' to (21),

³ Here the situation is different than that encountered in finding the limiting value of C_{f_0}' . There, k introduced a singularity into C_{f_0}' at its limiting value and we required the precise direction of approach to the limiting value.

Fig. 3. Limiting value of $C_{f_0}' - C_{f_0}$.Fig. 4. Typical curves of ΔC_{f_0} .

$$\Delta C_{f_0} = \bar{C}_{f_0}' - \bar{C}_{f_0} + \frac{W_0}{B_0 - T_0} + \frac{2}{\pi} \left\{ \log \frac{B_0 - T_0}{W_0} + \log \frac{k_0 K'}{2\pi} \right\} + O(g). \quad (22)$$

Accordingly, since $\bar{C}_{f_0}' - \bar{C}_{f_0} \rightarrow 0$ when $g \rightarrow 0$, as we have previously seen, in the limit,

$$\bar{\Delta C}_{f_0} = \frac{K'}{K} + \frac{2}{\pi} \log \frac{kK}{2\pi} \quad (23)$$

where k , K , and K' are defined in (14).

A curve expressing the relationship between $\bar{\Delta C}_{f_0}$ and $W_0/(B_0 - T_0)$ is presented in Fig. 3. When $W_0/(B_0 - T_0) \rightarrow 0$, $K \rightarrow \infty$ and $k \rightarrow 1$ so that $\bar{\Delta C}_{f_0}$ becomes infinite logarithmically. On the other hand, when $W_0/(B_0 - T_0) \rightarrow \infty$, $K' \rightarrow \infty$ and $k \rightarrow 0$ so that $\bar{\Delta C}_{f_0} \rightarrow 0$.

For any given value of W_0 , B_0 , and T_0 , $\Delta C_{f_0} < \bar{\Delta C}_{f_0}$ so that values taken from Fig. 3 put an absolute upper limit on the interaction between the symmetrical odd-mode fringing capacitances. Fig. 4 gives two curves of ΔC_{f_0} versus S to illustrate how the general curve can be accurately deduced from information now at hand. The plotted points on the curves between the points marked with circles and triangles were obtained from the expression $\Delta C_{f_0} \approx C_{f_0}' - \bar{C}_{f_0}' + W_0/(B_0 - T_0) - T_0/S_0$, which follows from (13) when $\bar{C}_{f_0}' \approx \bar{C}_{f_0}$. This approximation gives the rapidly changing portion of the curves with great accuracy. When the approximate values were compared with the exact values marked with circles (from [2]), where they are least accurate, it was found that for $T = 0.4$ the error was of the order of 5×10^{-4} and for $T = 0.6$ the error was less than 1×10^{-5} . The values plotted at $S = 2$ were obtained from formulas given by Bates [4]. They are actually values for $S = \infty$. It is known however that the presence of the side walls this far from the inner conductor will have a negligible effect on C_{f_0}' and C_{f_0} . Thus the effect on ΔC_{f_0} will be even less.

REFERENCES

- [1] W. J. Getsinger, "Coupled rectangular bars between parallel plates," *IRE Trans. Microwave Theory Tech.*, vol. MTT-10, pp. 65-72, Jan. 1962.
- [2] H. J. Riblet, "The exact dimensions of a family of rectangular coaxial lines with given impedance," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 538-541, Aug. 1972.
- [3] F. Bowman, *Introduction to Elliptic Functions with Applications*. New York: Dover, 1961, pp. 83-84.
- [4] R. H. T. Bates, "The characteristic impedance of the shielded slab line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 28-33, Jan. 1956.

Swept Frequency Impedance Indicator Using Directional Couplers

SHUNICHIRO EGAMI

Abstract—A swept frequency impedance indicator which consists of directional couplers and detectors is described. Experimental equipment was made at a 17.5–19.5-GHz band, and successfully operated.

INTRODUCTION

Swept frequency impedance measurement is very useful in the design and analysis of microwave devices. The method described here is intended for swept frequency impedance measurement in the millimeter waveband.

Several impedance-indicating methods applicable to the millimeter waveband are frequency dependent inherently [1], [2], or magic T with relatively narrow bandwidth were used [3]. The method described here has no frequency dependence on its operation and the swept frequency band is limited by the bandwidth of directional couplers, detectors, and circulators, if used. Frequency of measurement can be raised easily to the millimeter waveband because couplers and detectors can operate equally at this band.

CIRCUIT FOR THE MEASUREMENT

The coupled output of a directional coupler has a 90° phase difference with the uncoupled output. This phase shift depends on the directivity of the coupler and is independent of the frequency. This property is applied in this method. Fig. 1 shows the configuration for swept frequency measurement. C3 is the power divider which splits the input power with negligible frequency response. C4 is the coupler which makes the 90° phase difference to get voltages proportional to $r \cos \phi$ and $r \sin \phi$ (r, ϕ : amplitude and phase of the reflection coefficient of the device under test). In this configuration the length of line C3–C6 and C3–L2–C5 must be the same. This is adjusted by the phase shifter or line stretcher L2. (This adjustment can be cor-

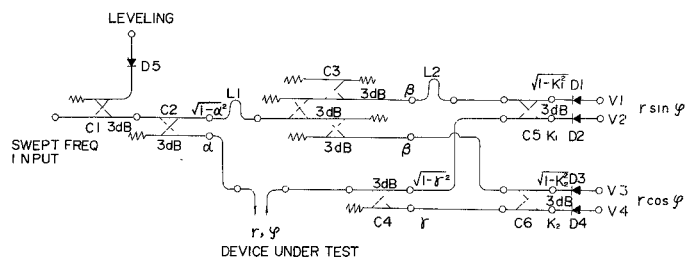


Fig. 1. Circuit for swept frequency measurement.

rectly carried out by connecting a short-ended long line as a device under test, and making the locus on the CRT circle.)

Then, changing the length of line L1, the measurement reference plane can be changed as desired. If the magnitude of the coupling of the couplers is designated as in Fig. 1, differences in the output voltage are given as follows.

$$\begin{aligned} V_1 - V_2 &= (k_1^2 - \frac{1}{2}) \{ r^2 (1 - \alpha^2) \beta^2 - \alpha^2 (1 - \gamma^2) \} \\ &\quad + 2\alpha \sqrt{1 - \alpha^2} \cdot k_1 \sqrt{1 - k_1^2} \cdot \sqrt{1 - \gamma^2} \cdot \beta \cdot r \cos \phi \quad (1) \\ V_3 - V_4 &= (k_2^2 - \frac{1}{2}) \{ r^2 (1 - \alpha^2) \beta^2 - \alpha^2 \gamma^2 \} \\ &\quad + 2\alpha \sqrt{1 - \alpha^2} \cdot k_2 \sqrt{1 - k_2^2} \cdot \gamma \cdot \beta \cdot r \sin \phi. \quad (2) \end{aligned}$$

If $k_1 = k_2 = 1/\sqrt{2}$ and α, β, γ are constant over the swept frequency band, voltage proportional to $r \cos \phi$ and $r \sin \phi$ can be obtained.

ERROR CONSIDERATION

Amplitude of the reflection coefficient on the CRT may be affected by:

- 1) nonuniform power level of the swept frequency input;
- 2) nonuniform frequency response of the detector;
- 3) nonuniform frequency response of the coupler.

The effect of 1) and 2) can be nullified by "leveling" of the sweep oscillator output using a 3-dB coupler (C1) and detectors (D1~D5) with matched characteristics.

Since the 3-dB coupler has reverse frequency response at the coupled and uncoupled output, the effect of 3) on C2, C5, and C6 can be neglected, as understood by (1) and (2). Frequency response of C3 can also be nullified using the configuration of Fig. 1. Eventually, the effect of 3) can be restricted to C4. So, care must be taken to get a flat frequency response at C4. (Change of radius is 1.2 percent/0.1 dB.) Position of the locus on the CRT is determined by the first terms of (1) and (2). Deviation of k_1 and k_2 from $1/\sqrt{2}$ makes the small shift of the locus by the frequency. So, care must be taken to make k_1 and k_2 equal to $1/\sqrt{2}$. In this method, correctness of the phase measurement is determined by the phase error of the couplers.

This can be very small if the directivity of the couplers is sufficiently high, and mismatch of the circuit is made small. Phase error of the coupler at the matched condition is given by the following equation.

$$|\delta\phi| < \frac{1}{2} \cdot \frac{1}{1 - k^2} \cdot \frac{1}{D}$$

where

- $\delta\phi$ phase error from $\pi/2$ (radian);
- k magnitude of coupling;
- D directivity of the coupler.

If $D = 35$ dB, $\delta\phi$ does not exceed 0.015° . But this may be degraded by mismatch of the circuit. Since the VSWR of the millimeter-waveband detectors is high, it is necessary to lower this about 1.2 to get phase error smaller than 1° [4].

RESULT OF EXPERIMENT

The configuration shown in Fig. 1 was constructed for the 18-GHz band. Directivity of the coupler was over 35 dB in the 15–21-GHz band. A circulator, one port of which was connected by a movable short, was used as the line stretcher L1. L2 was a simple phase shifter fixed at a suitable point. Reflection from the device under test can be derived by a circulator or a coupler. In this case, a circu-